A Multiple Constraint Approach for Finite Element Analysis of Moment Frames with Radius-cut RBS Connections

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ABSTRACT

After the 1994 Northridge earthquake it was evident that the traditional steel moment frames were not as ductile as previously presumed. Extensive research was subsequently undertaken to come up with an improved beam-column connection detail. Among the new and pre-qualified connection details is the Reduced Beam Section (RBS) connection in which a portion of the beam flange is removed near the connection area in an effort to decrease the stress and strain demands in the connection area. Currently, there is no accurate and efficient method for the analysis of moment frames with RBS connections. This study attempts to develop a new efficient and accurate linear elastic analysis method for moment frames with RBS connections. The method is based on mixed dimensional finite element analysis using multi-point constrain equations. The multi-point constraint equations are derived by equating the work done on either side of the mixed dimensional interface. The method has been applied to specific examples using the general purpose finite element program ANSYS. It has been found that the mixed dimensional models based on the derived constraint equations provide an acceptably accurate and efficient analysis method of moment frames with RBS connections.

Keywords: *Reduced Beam Section Connections, Multi-point Constraint Equations, Mixed Dimensional Modeling, Finite Element Method, Northridge Earthquake.*

1. INTRODUCTION

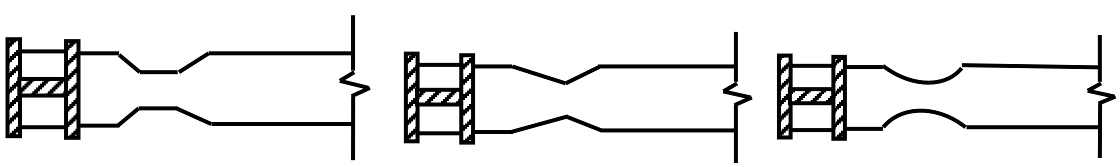
The Northridge Earthquake of Jan. 17, 1994 was milestone with respect to seismic design of steel structures in that it showed that the conventional beam-column connection in steel moment frames was not as ductile as previously believed [1,2]. Even though no total collapse occurred, investigations after the earthquake revealed a number of brittle fractures in the connection which could have resulted in total collapse had the earthquake lasted a little longer. The traditional moment connection, referred to as the pre-Northridge connection, typically consists of a bolted beam web and full penetration welds from the beam flanges to the column flange. In view of the non-ductile nature of the pre-Northridge connection, an emergency withdrawal of the building code provisions for the pre-qualified, pre-Northridge connection was put to effect in Sept. 1994.

After the Northridge earthquake, an extensive research was undertaken primarily under the umbrella of the SAC joint venture. (SAC is an acronym for the three participating organizations, namely, the Structural Engineers Association of California, the Applied Technology Council, and the Consortium of Universities for Research in Earthquake Engineering.) The aim of the research was to find out the reason for the poor performance of the pre-Northridge connection, and to come up with new and improved connection details. The poor performance of the standard connection was found to be a result of a number of complex and inter-related factors. The major factors include the geometry induced severe stress concentration, prevention of material yielding due to triaxial state of stress, poor welding and inspection due to connection geometry, increase in steel section sizes and changes in steel production over the years.

After years of research the SAC joint venture produced a number of new pre-qualified connection types, and one of these connection details is the Reduced Beam Section (RBS) connection.

2. REDUCED BEAM SECTION (RBS) CONNECTIONS

In the reduced beam section (RBS) connection, a portion of the beam flange is removed near the connection area. This will effectively make the connection area stronger than the beam, and thus decrease the stress and strain demands on the connection during strong earthquakes. Plastic hinges are expected to be formed in the reduced beam section where the ductility potential is very large. The basic idea of the RBS being reducing beam section near connection, the actual shape of the cut may be constant-cut, tapered-cut or radius-cut (See **Fig. 1**).



(a) Constant Cut RBS

(b) Tapered Cut RBS

(c) Radius Cut RBS

Fig. 1 Types of Reduced Beam Section (RBS) Connections

Extensive research on RBS connections shows that they have highly improved performance over the pre-Northridge connection [3]. Researches have also demonstrated the superiority of the radius-cut RBS connection, since with angular cuts cracks would tend to develop when the beams are subjected to large forces.

It is clear that the presence of the RBS will affect the structural response (stresses and displacements) of moment frames with RBS connections, and this has to be taken into account in the design process. In the existing practice, the RBS is accounted for by increasing the elastic drift in the absence of RBS by 9% corresponding to a flange reduction of 50 % [4]. (Percent flange reduction is computed between the original flange width and the smallest flange width in the RBS connection.) For lesser values of beam flange reductions, the drift amplification factor is obtained by linear interpolation. It has been reported that this simple approach does not necessarily give conservative results [5]. Thus there is a need for a more accurate method for the linear elastic first order analysis of moment frames with RBS connections. This study attempts to develop a new efficient method for the analysis of moment frames with RBS using mixed dimensional finite element modeling via constraint equations.

3. MIXED DIMENSIONAL FEM MODELING USING CONSTRAINT EQUATIONS

In most practical problems there area areas of the model that are an ideal candidate for dimensional reduction, but are bounded by areas that contain complex configurations such as discontinuities in geometry, loading, boundary conditions or material behavior. In many finite element analyses, reduced element types thus combined with higher dimensional element types in a single model. Long slender regions can be represented appropriately using beam elements, and thin zones can usually be modeled using shell elements, and thick portions are best represented as solid elements. However, practical models usually contain a mixture of more than one of the above regions. Thus, some sort of coupling scheme is required to form a link between the meshes of different types.

Mixed dimensional coupling using constraint equations has been shown to give good results [6]. In this approach, the transition between the different element types is achieved by multi-point constraint equations derived by equating the work done on either side of the interface.

In this study, moment frames with RBS connection has been treated as mixed models involving beam and shell elements. The connection zone including the RBS is modeled by shell elements while the portion of the beams and columns away from the connection area is modeled by beam elements. Coupling is achieved by multi-point constraint equations derived by equating the work done on the beam and shell side of beam-shell interface.

4. BEAM-SHELL COUPLING USING CONSTRAINT EQUATIONS

The general method of coupling of elements of dissimilar dimension via constraint equations has been developed by McCune and Monaghan [6]. The method is based on equating the work done on both sides of the interface of different elements. A basic requirement of the method is the knowledge of the stress distribution at the interface which can be obtained from the results of beam theory. The derivation of the constraint equations for beam-shell coupling will be illustrated for axial force as follows.

Beam-Shell Coupling: Axial Force

The mixed beam-shell model is shown in **Fig. 2**. The aim is to couple the axial displacements of the shell nodes at the transition to the axial displacements of the beam nodes such that the distribution of stress on the interface is similar to that given by elastic theory.

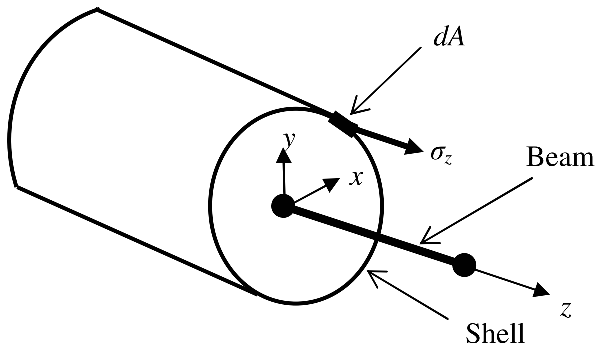


Fig. 2 Beam-Shell Coupling

Under the influence of axial force F_z alone, the only non-zero stress on the shell side of the direct stress σ_z. Equating the work done by the 1D beam with the work done by the surface stress of the shell, the following equation results.

$$F_z w = \int_A \sigma_z W dA \qquad \text{... (1)}$$

where w is the beam node displacement, W is shell node displacements.

If the shell region is slender, then the axial stress is uniform over the cross-section and is given by:

$$\sigma_z = \frac{F_z}{A} \qquad \text{... (2)}$$

where F_z is the axial force, and A is the cross-sectional area. In the finite element model, the axial displacements at any point is found by interpolation from nodal displacements.

$$W = [N]\{W\} \qquad \text{... (3)}$$

where $[N]$ is the shape function, and $\{W\}$ is nodal displacements.

Substituting Eq. 2 and Eq. 3 into Eq. 1, we find the following.

$$F_z w = \frac{F_z}{A} \sum_{i=1}^{Nelements} \int_{A_i} [N] dA \{W\} \qquad \text{... (4a)}$$

$$w = \frac{1}{A} \sum_{i=1}^{Nelements} \int_{l_i} t [N] dl \{W\} \qquad \text{... (4b)}$$

where t is the shell thickness and l is the element edge length. Since the cross-sectional area is simply the product of the edge length and shell thickness, Eq. 4b can be written as:

$$\left(\sum_{j=1}^{Nelements} t_j l_j \right) w = \sum_{i=1}^{Nelements} \int_{l_i} t [N] dl \{W\} = [B]\{W\} \qquad \text{... (5)}$$

where $[B]$ is a matrix of constants depending on the cross-sectional parameters and shape functions. Thus, we arrive at a multi-point constraint equation of beam-shell coupling for axial force of the following form.

$$-a_o w + B_1 W_1 + B_2 W_2 + B_3 W_3 + \ldots = 0 \qquad \text{... (6)}$$

where a_o, B_1, B_2, ... are constants depending on the cross-section and type of shape functions.

Similarly, constraint equations for bending moments, shear forces, and torsion can be derived. It should be emphasized that the constraint equations are derived based on an assumed stress distribution at the interface. Thus, they are only as good as the accuracy of the assumed stress distribution.

Beam-Shell Model of Moment Frames with RBS Connections

In this study moment frames with RBS connections are modeled as mixed beam-shell models: the connection area using shell elements and other parts using beam elements. (See **Fig. 3**). A four node quadrilateral isoparametric shell element has been used for the shell element. Constraint equations are derived for axial force (in the z-direction), bending moments about the x- and y-axes, and shear forces in the y and x directions. The stress distributions due to bending moments and shear force have been assumed in accordance with simple beam theory. A computer program has been written to generate constraint equations for axial force, shear forces and bending moments for a given I-section.

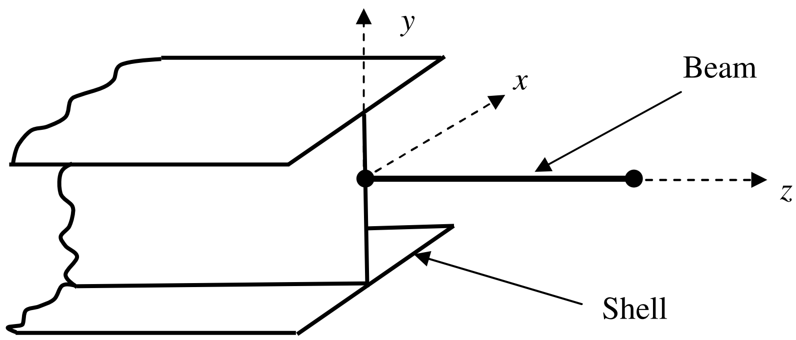


Fig. 3 Beam-Shell Model of Moment Frames with RBS

5. APPLICATION OF CONSTRAINT EQUATIONS

Once the constraint equations have been determined, they are incorporated in the finite element formulation by standard methods such as Lagrange multiplier or penalty function methods. In this study the constraint equations are implemented using the general purpose finite element program ANSYS [7]. To illustrate the validity of the constraint equations, a cantilever beam has been considered.

Cantilever Beam

The first example considered is the cantilever beam shown in **Fig. 4**. The following geometric and material data has been considered.

Length = 4.0 m

Beam Cross-section: $h = 533\ mm,\ b_f = 330\ mm,\ t_f = 22.0\ mm,\ t_w = 13.4\ mm$

Material: $E = 200\ GPa\ ,\ \upsilon = 0.3$

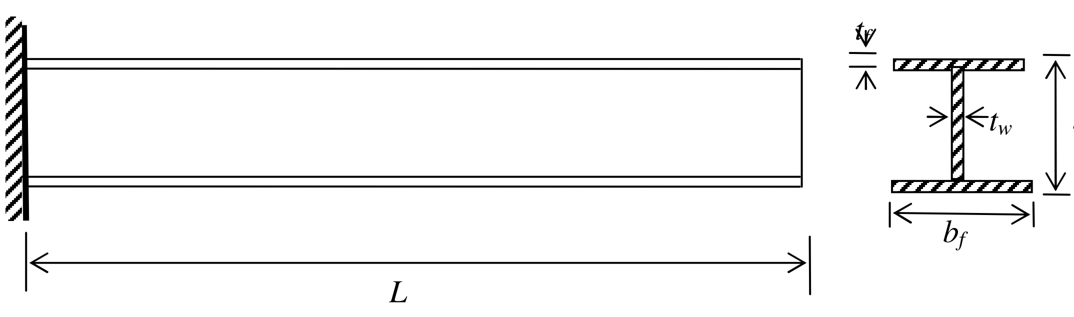
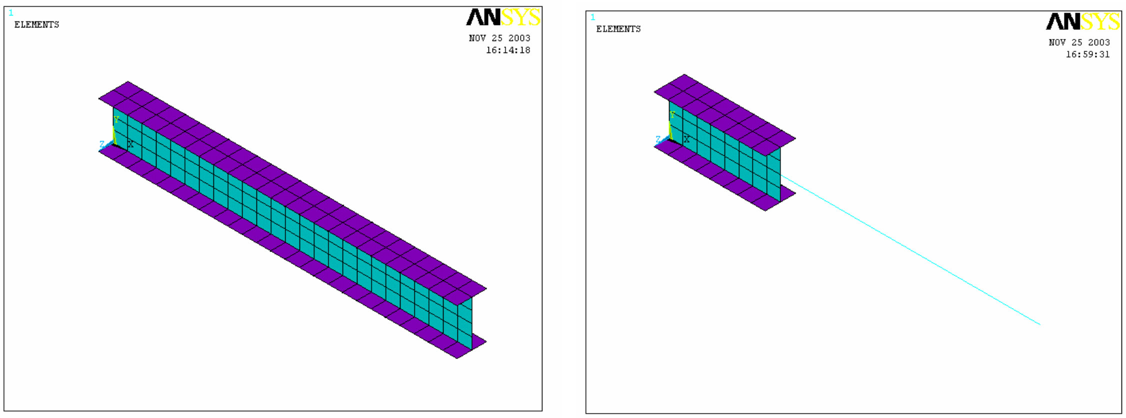


Fig. 4 Cantilever Beam

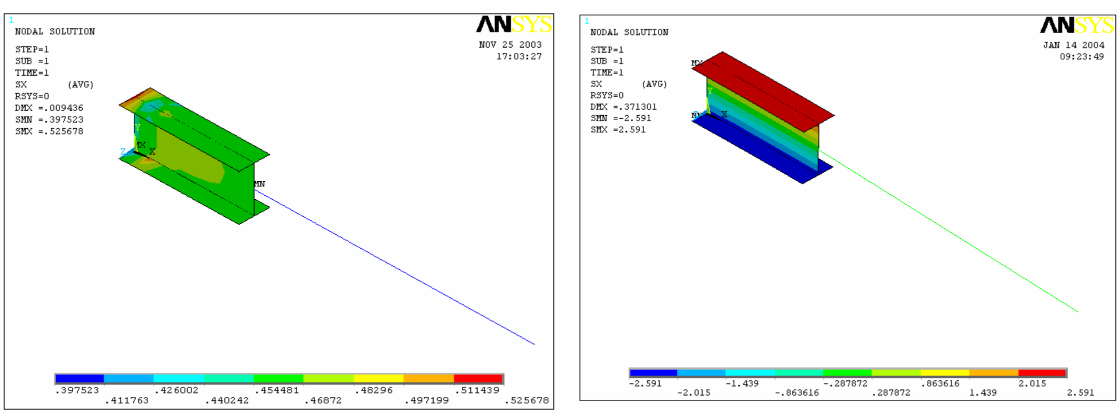
The cantilever is initially considered without RBS and has been modeled as a full shell model as well as a mixed beam-shell model with varying extent of the shell region. Three different load cases have been considered: an axial tensile force of 10 kN, a transverse shear force of 10 kN, and a bending moment of 10 kN.m applied at the free end. The full shell and mixed models of the cantilever beam without RBS are shown in **Fig. 5**.



(a) Full Shell Model (b) Mixed Beam-Shell Model

Fig. 5 Full Shell and Mixed Model of Cantilever Beam

The full shell and mixed models have been subjected to the three load cases mentioned above. It has been found that the derived constraint equations result in a smooth stress transition at the interface similar to that given by elastic theory. The stress contour showing the results of the mixed model under different loads is shown in **Fig. 6**.



(a) Axial Force (b) Bending Moment

Fig. 6 Normal Stress Distribution due to Axial Force and Bending Moment

A comparison of displacements and stresses between the full shell and mixed models at the mixed dimensional interface has shown that the results of the mixed model are in a very close agreement with that of the full shell model. For axial force and bending moment, the maximum percent error of displacement and stresses of the full shell and mixed models is found to be less than 2 %. For transverse load, the maximum percent error on displacement is less than 1 % and the maximum percent on normal stresses is less than 7 %. **Table 1** gives a summary of the tip deflections of the cantilever beam as given from the beam, full shell and mixed models for the different load cases. The results for the beam model are determined from simple beam theory including shear deformations. It can be seen that the mixed models give results of tip deflection in very close agreement with the full shell model.

Table 1: Tip Deflection of Cantilever Beam without RBS

Model	Shell Region (% of Beam Length)	Number of Shell Elements	Axial Tip Deflection (mm) (due to axial load of 10kN)	Transverse Tip Deflection (mm) (due to a couple load of 10kN.m)	Transverse Tip Deflection (mm) (due to a transverse load of 10 kN)
Beam	0	0	0.0094910	0.37070	1.01816
Mixed	30	64	0.0094358	0.37130	1.02690
Shell	100	200	0.00937660	0.36424	1.04642

Next the cantilever is considered with RBS data as shown in **Fig. 7**. The cantilever with RBS is modeled as full shell model and mixed beam-shell models as shown in **Fig. 8**.

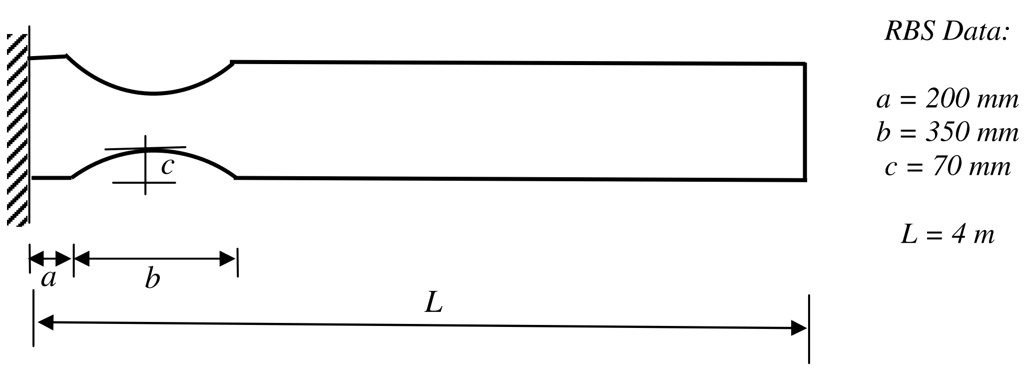
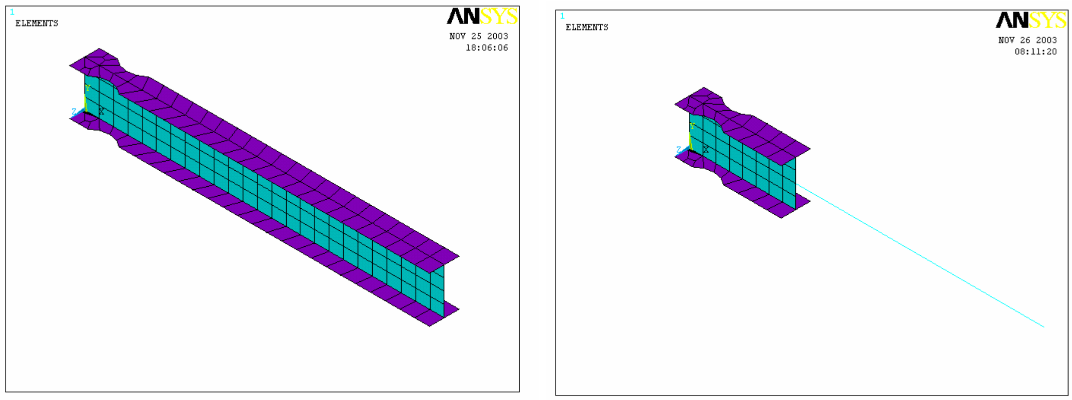


Fig. 7 Cantilever Beam with RBS



(a) Full Shell Model

(b) Mixed Beam-Shell Model

Fig. 8 Full Shell and Mixed Model of Cantilever Beam with RBS

It is again observed that the mixed models based on the derived constraint equations result in a smooth transition of stress at the mixed dimensional interface. The tip deflections corresponding to the full shell and mixed models are given in **Table 2**. It is noted that the mixed models provide results of tip deflection in close agreement with that of the full shell model.

Table 2: Tip Deflection of Cantilever with RBS

Model	Shell Region (% of Beam Length)	Axial Tip Deflection (mm) (due to axial load of 10kN)	Transverse Tip Deflection (mm) (due to couple load of 10kN.m)	Transverse Tip Deflection (mm) (due to transverse load of 10kN)
Mixed	20	0.0097019	0.39527	1.1123
Mixed	30	0.0096817	0.39459	1.1099
Mixed	40	0.0096716	0.39442	1.1090
Mixed	50	0.0096548	0.39285	1.1027
Shell	100	0.0096222	0.38719	1.1289

6. CONCLUSION

An efficient and accurate analysis method for the analysis of moment frames with RBS has been presented. The method relies on the development of multi-point constraint equations by equating the work done on either side of a mixed dimensional interface. The resulting stress distribution is in agreement with that given by elastic theory. It has been shown that mixed dimensional modeling using multi-point constraint equations provides an acceptably accurate and efficient method for the elastic analysis of moment frames with RBS. The mixed dimensional model provides a more realistic representation of the joint area. Unlike the frame model in which joints are commonly assumed to be rigid, the mixed dimensional model considers deformations within the connection. It has been shown that frame model with rigid joints may significantly underestimate deflections unless the joint has been properly detailed so as to validate the rigid-joint assumption. The mixed model, on the other hand, considers the actual deformations in the joint region, and thus yields more realistic deflections corresponding to a given joint detail.

7. RECOMMENDATIONS

In the present study it has been shown that mixed dimensional analysis using multi-point constraint equations provides an efficient and accurate analysis of moment frames with RBS. However, the work is not exhaustive and there are a number of avenues for improvement and further investigation such as those given below. Multi-point constraint equations are dependent on the element type. In this study constraint equations are derived for a four node quadrilateral shell element. Constraint equations for other types of shell elements can be developed using the same approach. The length over which the shell region extends from the connection is an important one which should be answered in such a way as to balance accuracy and computation. Further study to systematically arrive at an optimum length of shell region is necessary. Finally, a self contained program for the analysis of moment frames with RBS connection using multi-point constraints can be considered as an area of further work.

8. REFERENCES

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